Further Studies on a Melting Problem with Natural Convection

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An approximate mathematical solution to the solid-liquid moving boundary problem, including buoyancy force, was made by Tien and Yen (*3*). In that study, the classical Neumann problem (*1*) was extended to include the effect of natural convection in the liquid phase caused by the melting of ice from below. In a recent paper, Yen et al. (*5*) conducted an experimental investigation of the same problem, aimed at confirming their theoretical study. The experimental data were found to be in qualitative agreement with the results predicted from their solutions. The purpose of this communication is threefold: to present new data covering a wider range of experimental parameters; to modify the analytical analysis reported in reference *3*; and to extend the O'Toole and Silveston (*2*) correlation to the case involving phase change.

From the previous study (*5*), it was proposed that the correlation by O'Toole and Silveston (*2*) of natural convection heat transfer for fluids confined between two horizontal plates be extended to incorporate the dimensionless thermal parameters ϕ and $R_{\Delta T}$. To facilitate the mathematical analysis in this paper, an arbitrary term B, instead of 0.305, will be used as the exponent to the Rayleigh number in the heat transfer expression, which becomes

$$N_{Nu} = \frac{hS}{k_1} = 0.104\,(N_{Pr})^{0.084}(N_{Ra})^{B} \tag{1}$$

Following the same analysis as the previous investigation (*3*), the dimensionless heat flux prescribed to the water-ice interface becomes

$$H^+(t) = R_{\Delta T}\frac{k_1}{k_2}\,\sigma^{-(1-3B)} \tag{2}$$

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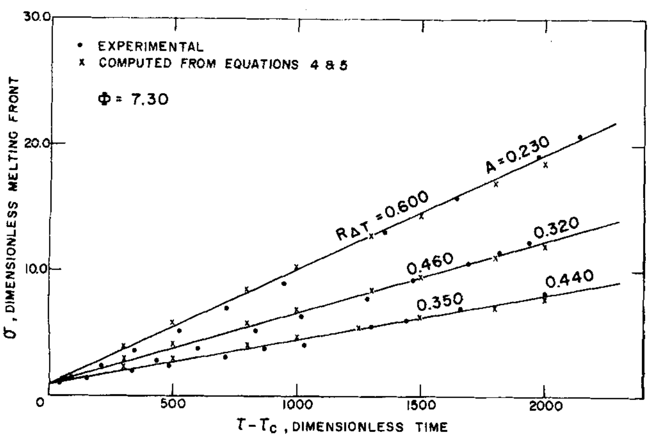

Fig. 1*a*. Relation between dimensionless melting front σ and dimensionless time $\tau - \tau_c$. $\phi = 7.30$.

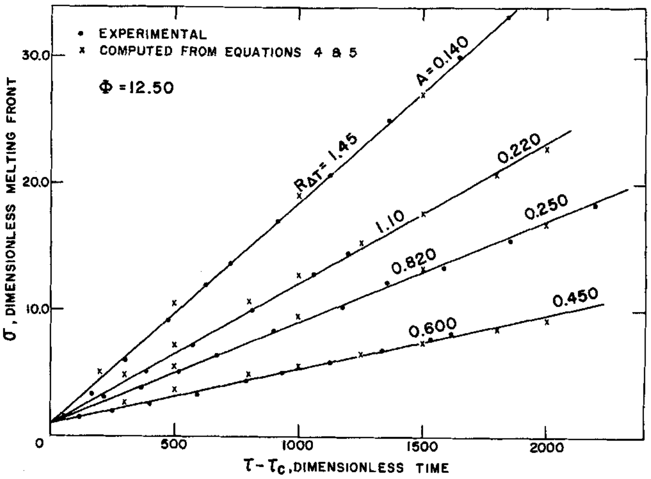

Fig. 1*b*. Relation between dimensionless melting front σ and dimensionless time $\tau - \tau_c$. $\phi = 12.50$.

or

$$H^+(t) = R_{\Delta T}\frac{k_1}{k_2}\,\sigma^{-A} \tag{3}$$

The dimensionless melting front σ can be obtained by solving the following pair of first-order nonlinear differential equations. The only difference from the corresponding equations in reference 3 as corrected in reference 5 is that an unknown term A has replaced 0.085.

$$\frac{d\eta}{d\tau} = \frac{4}{\phi}\left[\frac{3(1+\phi)}{\eta} - R_{\Delta T}\sigma^{-A}\left(\frac{k_1}{k_2}\right)\right] \tag{4}$$

$$\frac{d\sigma}{d\tau} = \frac{1}{\phi}\left[R_{\Delta T}\,\sigma^{-A}\left(\frac{k_1}{k_2}\right) - \frac{3}{\eta}\right] \tag{5}$$

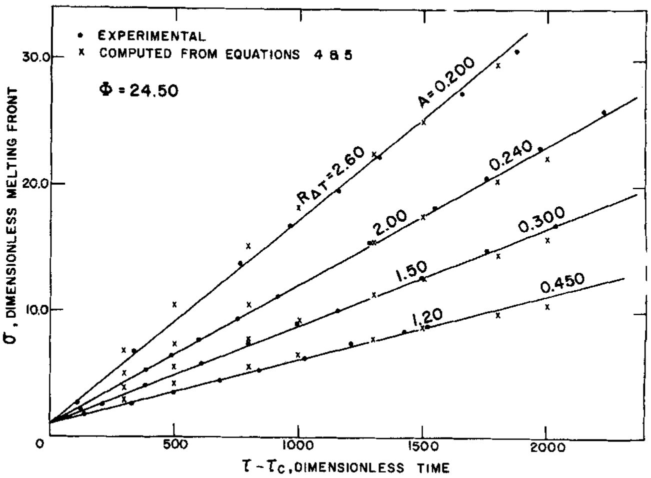

Fig. 1*c*. Relation between dimensionless melting front σ and dimensionless time $\tau - \tau_c$. $\phi = 24.50$.

Table 1. Summary of Experimental Parameters and Values of A

Experiment No.	T_o, °C.	T_s, °C.	ϕ	$R_{\Delta T}$	A
1	−22.00	7.72	7.30	0.350	0.440
2	−22.00	10.05	7.30	0.460	0.320
3	−22.00	13.10	7.30	0.600	0.230
4	−13.00	7.72	12.50	0.600	0.450
5, 6	−13.00	10.60	12.50	0.820	0.250
7	−13.00	14.02	12.50	1.100	0.220
8, 9	−13.00	18.80	12.50	1.450	0.140
10	−6.50	7.75	24.50	1.200	0.450
11	−6.50	9.83	24.50	1.500	0.300
12, 13	−6.50	13.00	24.50	2.000	0.240
14, 15	−6.50	18.00	24.50	2.600	0.200

The experimental apparatus and procedure are described in detail in a paper by Yen et al. (5). In all experiments, bubble-free homogeneous ice samples were prepared before the melting experiment. Caution was always taken to eliminate any possible entrainment of air during the assembly operation. This is necessary in order to get reliable and reproducible results. Figures 1*a*, 1*b*, and 1*c* are plots of dimensionless melting front σ vs. dimensionless time $\tau - \tau_c$. Note that when $\tau - \tau_c = 0$, $\sigma = 1$. In the figures the black dots represent the experimental data. Equations (4) and (5) were integrated simultaneously by the method of Runge-Kutta. For each pair of $R_{\Delta T}$ and ϕ, a trial and error procedure was used to find a specific value of A that gave the best fit to the experimental data of the corresponding set values of $R_{\Delta T}$ and ϕ. The computed results are indicated by crosses along with the values of A in Figures 1*a*, *b*, and *c*. Table 1 shows the summary of the experimental parameters and the values of A. ϕ was calculated using the specific heat of ice C_{p2} at the mean value of initial and melting temperatures (4). Latent heat of fusion L was taken to be 80 cal./g. It should be noted that experiments 5, 8, 12, and 14 were conducted under identical conditions with 6, 9, 13, and 15, respectively. From Figures 1*a*, 1*b*, and 1*c* it can be seen that the computed results have slightly higher values than the experimental data for the small values of $\tau - \tau_c$, while for large values the computed results are slightly lower than the experimental data. However, in general, the results from theory and experiment are in close agreement.

It can be observed from Figures 1*a*, 1*b*, and 1*c* that for a specific value of ϕ, the value of A increases as $R_{\Delta T}$ decreases. For the same $R_{\Delta T}$, values of A increase as ϕ increases (see Figures 1*a*, 1*b*, and 1*c* for instance, when $R_{\Delta T} = 0.60$, $A = 0.230$ for $\phi = 7.30$ and $A = 0.450$ for $\phi = 12.50$). Therefore, a functional relationship among A, $R_{\Delta T}$, and ϕ can be expressed as

$$A = a\ \phi^m (R_{\Delta T})^n$$

where constant a and exponents m and n are to be determined. From a log-log plot of A vs. $R_{\Delta T}$ with ϕ as parameter, the values of A for different ϕ fall into parallel linear lines, the slope of the lines or the value of n is found to be −1.2. A similar log-log plot of $A(R_{\Delta T})^{1.2}$ vs. ϕ also give a linear relationship. The slope or the value of m is 1.2. The intercept or the value of a is 0.0114. Thus the value of A is given by the expression

$$A = 0.0114\ (\phi/R_{\Delta T})^{1.2} \tag{7}$$

Since $A = 1 - 3B$ [Equations (2) and (3)]

$$B = \frac{1 - 0.0114\ (\phi/R_{\Delta T})^{1.2}}{3} \tag{8}$$

Substituting the expression for B into Equation (1), we arrive at the correlation for natural convection heat transfer for fluids being melted from below and confined by a rigid lower boundary.

By substituting Equation (8) into Equation (1), the final correlation which involved the Prandtl and Rayleigh numbers and thermal parameters ϕ and $R_{\Delta T}$ is obtained:

$$N_{Nu} = \frac{hS}{k_1} = 0.104\ (N_{Pr})^{0.084}\ (N_{Ra})^{[1-0.0114(\phi/R_{\Delta T})^{1.2}]/3} \tag{9}$$

The above expression applies to the water-ice system and is valid for ϕ varying from 7.30 to 24.50 and $R_{\Delta T}$ from 0.35 to 2.60. For ice not entirely free from air bubbles, the air will accumulate at the water-ice interface as the melting proceeds. The overall effect on the melting rate depends on the fraction of the interface covered by the air. Since the thermal properties of homogeneous ice are more or less isotropic, it is believed that the crystal structure or the manner in which the ice is frozen will have no effect on the macroscopic melting rate.

NOTATION

C_p = heat capacity
$H^+(t)$ = dimensionless heat flux given by Equation (2) or (3)
k = thermal conductivity
h = heat transfer coefficient
L = latent heat of fusion
N_{Nu} = Nusselt number
N_{Pr} = Prandtl number
N_{Ra} = Rayleigh number
$R_{\Delta T}$ = dimensionless parameter defined as $(T_s - T_m)/(T_m - T_o)$
S = melting front
S_c = transitional melting front
t = time
T_m = melting temperature
T_o = initial ice temperature ($T_o < T_m$)
T_s = warm plate temperature ($T_s > T_m$)

Greek Letters

σ = dimensionless solid liquid interface position defined as S/S_c
δ = dimensionless thermal boundary-layer thickness
η = $\delta - \sigma$
ϕ = dimensionless parameter defined as $L/C_{p2}(T_m - T_o)$
τ = dimensionless time defined as $\alpha_2 t/S_c^2$
τ_c = transitional dimensionless time

Subscripts

1 = liquid phase
2 = solid phase

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